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Polarization oscillations of near-field thermal emission

MANABU MACHIDA,[1] EVGENII NARIMANOV,[2] AND JOHN C. SCHOTLAND[3,*]

[1]Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109, USA
[2]Department of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47906, USA
[3]Department of Mathematics and Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA
*Corresponding author: schotland@umich.edu

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We consider the polarization of thermal emission in the near field of various materials, including dielectrics and metallic systems with resonant surface modes. We find that, at thermal equilibrium, the degree of polarization exhibits spatial oscillations with a period of approximately half the optical wavelength, independent of material composition. This result contrasts with that of Setala *et al.* [Phys. Rev. Lett. 88, 123902 (2002)], who find monotonic decay of the degree of polarization for systems in local thermal equilibrium.

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1. INTRODUCTION

Coherence theory is one of the cornerstones of optical physics [1]. Its goal is to describe the statistical regularities of electromagnetic fields in terms of field correlations and their relation to measurable quantities. Implicit in the formulation of coherence theory is the notion of a statistical ensemble of random fields. However, the origin of the randomness of the fields is not explicitly part of the theory. As a result, the theory of coherence is primarily concerned with the propagation of correlation functions, which has the powerful consequence that its predictions are, in some sense, independent of the underlying probability distribution of the fields.

A notable exception to the above remarks is provided by the theory of thermal emission of radiation [1,2]. In this context, the currents, which act as sources of the optical field, are taken to obey the fluctuation-dissipation theorem. Thus, it is possible to calculate the correlation function of the field in terms of the statistics of the current. It follows that the emitted field displays temporal and spatial coherence [3]. Moreover, the near- and far-field coherence functions manifest strikingly different behavior [4–8]. In particular, in materials that support resonant surface waves, the spectrum of emitted radiation changes qualitatively on propagation, ranging from extreme narrowing at subwavelength scales to broadband in the far field. Likewise, the spatial coherence of emitted light is dramatically modified in the near field, with a coherence length that is much smaller than the $\lambda/2$ far-field limit of blackbody radiation. In either case, the alteration in coherence is due to the decay of evanescent modes of the field on propagation.

The near-field polarization of thermal emission has also received attention [9,10]. It was found that, at *local* thermodynamic equilibrium, the emitted field becomes depolarized, with the degree of polarization decaying monotonically upon propagation into the far zone. By the term local, we mean that the medium is held at a nonzero temperature and radiates into vacuum at zero temperature. In this case, pure thermal emission occurs, with a net heat flux transferred from the material to vacuum. In contrast, we study the corresponding problem for systems in thermal equilibrium, where the net flux vanishes. Instead of monotonic decay, we predict that the degree of polarization exhibits spatial oscillations with a period of approximately $\lambda/2$. We illustrate this result for several materials, including lossless dielectrics and metallic systems with resonant surface plasmon modes. We note that the presence of polarization oscillations was observed numerically by Dorofeyev and Vinogradov [11]. However, these authors did not provide a mathematical analysis of the phenomenon nor did they predict the universal oscillatory behavior of the polarization.

The remainder of this paper is organized as follows. In Section 2, we recall some basic facts from coherence theory. We also obtain the expression for the degree of polarization in the half-space geometry at thermal equilibrium. In Section 3, we calculate the polarization for several materials, elucidate the presence of oscillations and provide an analysis of the distance-dependence of the degree of polarization. Our conclusions are presented in Section 4, and Appendix A presents details of the mathematical analysis.

2. COHERENCE AND POLARIZATION

The fundamental quantity of coherence theory in the space-frequency domain is the cross-spectral density W_{ij}, which is defined by

$$W_{ij}(\mathbf{r},\mathbf{r}';\omega)\delta(\omega-\omega') = \langle E_i(\mathbf{r},\omega)E_j^*(\mathbf{r}',\omega')\rangle. \quad (1)$$

Here, $\mathbf{E}(\mathbf{r},\omega)$ is the electric field at the position $\mathbf{r}$ and frequency ω, the presence of the delta function indicates that the field is taken to be statistically stationary and $\langle\cdots\rangle$ denotes the ensemble average. We note that $W_{ij}(\mathbf{r},\mathbf{r}';\omega)$ for $\mathbf{r}\neq\mathbf{r}'$ is a measure of the spatial coherence of the electric field. The degree of coherence is defined to be [12,13]

$$\mu^2(\mathbf{r},\mathbf{r}';\omega) = \frac{\mathrm{Tr}[W(\mathbf{r},\mathbf{r}';\omega)W(\mathbf{r}',\mathbf{r};\omega)]}{\mathrm{Tr}W(\mathbf{r},\mathbf{r};\omega)\mathrm{Tr}W(\mathbf{r}',\mathbf{r}';\omega)}. \quad (2)$$

It can be seen that $0\leq\mu\leq 1$. The case $\mu=0$ corresponds to an incoherent field and $\mu=1$ to a coherent field; otherwise, the field is said to be partially coherent.

There is a fundamental link between polarization and coherence. If we consider $W_{ij}(\mathbf{r},\mathbf{r}';\omega)$ for $\mathbf{r}=\mathbf{r}'$, then W is a measure of the polarization of the field. The degree of polarization P is defined as [9,10,14,15]

$$P^2(\mathbf{r},\omega) = \frac{3}{2}\left[\frac{\mathrm{Tr}[W^2(\mathbf{r},\mathbf{r};\omega)]}{\mathrm{Tr}^2[W(\mathbf{r},\mathbf{r};\omega)]} - \frac{1}{3}\right]. \quad (3)$$

It can be shown that and $0\leq P\leq 1$. When $P=0$ the field is said to be unpolarized, and if $P=1$ the field is polarized; otherwise, the field is partially polarized.

We consider a system consisting of two homogeneous half-spaces, which are in thermal equilibrium at a temperature T. The lower half-space $z<0$ is taken to consist of a nonmagnetic lossy material with a generally complex dielectric permittivity $\epsilon_1(\omega)$. The upper half-space $z>0$ consists of a nonmagnetic material with a real and frequency-independent permittivity ϵ_2. In this setting, the fluctuation-dissipation theorem can be used to relate the cross-spectral density to the Green's tensor G_{ij} by the formula

$$W_{ij}(\mathbf{r},\mathbf{r}';\omega) = 2\pi k_0^2\hbar\coth\left(\frac{\hbar\omega}{2k_BT}\right)\mathrm{Im}G_{ij}(\mathbf{r},\mathbf{r}'), \quad (4)$$

where $k_0=2\pi/\lambda$ is the free-space wavenumber and k_B is Boltzmann's constant [16].

The Green's tensor obeys the equation

$$\nabla\times\nabla\times G(\mathbf{r},\mathbf{r}') - k_0^2\epsilon(z)G(\mathbf{r},\mathbf{r}') = 4\pi\delta(\mathbf{r}-\mathbf{r}')I, \quad (5)$$

where

$$\epsilon(z) = \begin{cases}\epsilon_1 & \text{if} \quad z<0,\\ \epsilon_2 & \text{if} \quad z>0,\end{cases} \quad (6)$$

and I is the unit tensor. The Green's tensor also obeys the boundary conditions

$$\hat{\mathbf{z}}\times G(\mathbf{r},\mathbf{r}')|_{z=0^+} = \hat{\mathbf{z}}\times G(\mathbf{r},\mathbf{r}')|_{z=0^-}, \quad (7)$$

$$\hat{\mathbf{z}}\times\nabla\times G(\mathbf{r},\mathbf{r}')|_{z=0^+} = \hat{\mathbf{z}}\times\nabla\times G(\mathbf{r},\mathbf{r}')|_{z=0^-}, \quad (8)$$

which correspond to the continuity of the tangential electric and magnetic fields, where G is interpreted as the electric field radiated by a point source.

It will prove useful to expand the Green's tensor into plane waves [17]:

$$G_{ij}(\mathbf{r},\mathbf{r}') = \int\frac{\mathrm{d}^2q}{(2\pi)^2}e^{i\mathbf{q}\cdot(\rho-\rho')}g_{ij}(z,z';\mathbf{q}), \quad (9)$$

where

$$g_{ij}(z,z';\mathbf{q}) = \frac{2\pi i}{k_{2z}}[(r_s\hat{s}_i\hat{s}_j + r_p\hat{p}_{+i}\hat{p}_{-j})e^{ik_{2z}(z+z')} + (\hat{s}_i\hat{s}_j + \hat{p}_{+i}\hat{p}_{+j})e^{ik_{2z}(z-z')}]. \quad (10)$$

Here,

$$\hat{\mathbf{s}} = \hat{\mathbf{q}}\times\hat{\mathbf{z}}, \qquad \mathbf{k}_\pm = \mathbf{q}\pm k_{2z}\hat{\mathbf{z}}, \qquad \hat{\mathbf{p}}_\pm = \hat{\mathbf{s}}\times\hat{\mathbf{k}}_\pm, \quad (11)$$

where $k_{2z}=\sqrt{\epsilon_2k_0^2-q^2}$. The Fresnel reflection coefficients are given by

$$r_s = \frac{k_{2z}-k_{1z}}{k_{2z}+k_{1z}}, \qquad r_p = \frac{k_{2z}\epsilon_1-k_{1z}}{k_{2z}\epsilon_1+k_{1z}}, \quad (12)$$

where $k_{1z}=\sqrt{\epsilon_1k_0^2-q^2}$.

3. APPLICATIONS

We now describe applications of the above results to various materials. In the cases considered, the system has the temperature $T=300$ K and the upper half-space is taken to have permittivity $\epsilon_2=1$. The optical properties of the materials that comprise the lower half-space are given in Table 1 [18]. In Fig. 1, we plot the degree of polarization P as a function of the distance z from the interface for several materials including glass, tungsten, silver, and silicon carbide (SiC). We see that the field is partially polarized near the interface and becomes depolarized at large distances. In Fig. 2, we plot the corresponding degree of coherence μ as a function of the transverse distance $\rho=|\mathbf{r}-\mathbf{r}'|$, for points $\mathbf{r}$ and $\mathbf{r}'$ in the plane $z=z_0$. As may be expected, the field is partially coherent near the interface and becomes incoherent with propagation, consistent with the results of [4].

Evidently, the behavior of both P and μ depends sensitively on the dielectric permittivity of the material under investigation. In the case of glass, which is a nonlossy dielectric at the wavelength $\lambda=500$ nm, the field is relatively less polarized. In contrast, silver exhibits a surface plasmon resonance at $\lambda=620$ nm, and the near-field is strongly polarized. This should be compared with the case of tungsten, which does not exhibit a plasmon resonance at $\lambda=500$ nm, where it can be seen that P is correspondingly reduced. Finally, we consider the case of SiC, which, at $\lambda=11.36$ μm, supports surface-phonon polariton modes. We see that, as in the example of silver, the near-field is strongly polarized.

Table 1. Optical Properties of Considered Materials

Material	λ	ϵ
Glass	500 nm	2.25
W	500 nm	$4.35+18.1i$
Ag	620 nm	$-15.0+1.0i$
SiC	11.36 μm	$-12.2+0.71i$

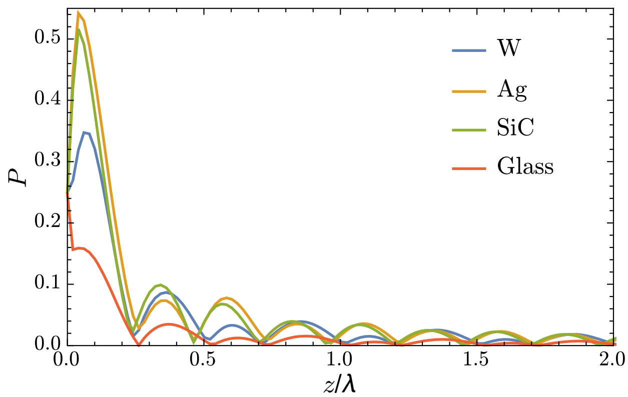

Fig. 1. Degree of polarization of glass, SiC, silver, and tungsten as a function of distance z from the interface.

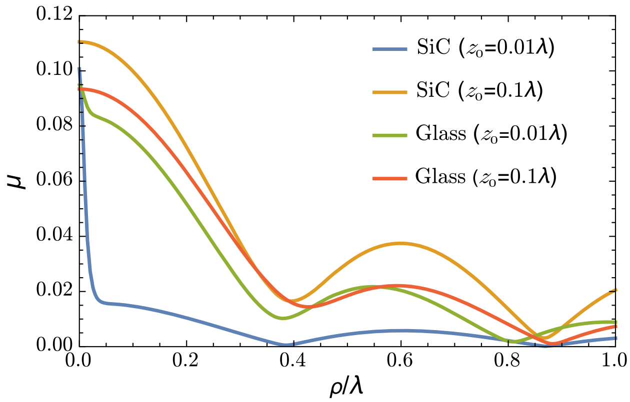

Fig. 2. Degree of coherence of glass and SiC as a function of transverse distance ρ in the plane $z = z_0$ at various distances z_0 above the interface.

A striking feature of Fig. 1 is the oscillatory nature of the distance-dependence of the degree of polarization. A straightforward asymptotic analysis of the integral Eq. (9) defining $\mathrm{Im}G_{ij}$ shows that for $z \gg \lambda$, the envelope of the oscillation decays as $P \sim 1/z$. Moreover, if $\mathrm{Re}(\epsilon_1) \ll -1$, it can be seen that

$$P \sim \frac{1}{2}\left|\frac{\sin(4\pi z/\lambda + 2/\sqrt{|\mathrm{Re}(\epsilon_1)|})}{4\pi z/\lambda + 2/\sqrt{|\mathrm{Re}(\epsilon_1)|}}\right|. \quad \textbf{(13)}$$

Thus, the period of the oscillations is asymptotically $\lambda/2$, independent of the material. In Fig. 3, we compare the above asymptotic formula with the exact result obtained from Eq. (3). As may be expected, there is excellent agreement for SiC and relatively poor agreement for glass.

It is important to note that polarization oscillations are not present for systems in local thermal equilibrium, where P decays monotonically after reaching a maximum at $z \lesssim \lambda$. [9]. This difference can be explained by the interference between modes in the upper half-space, a mechanism that is not present in the calculations presented in [9].

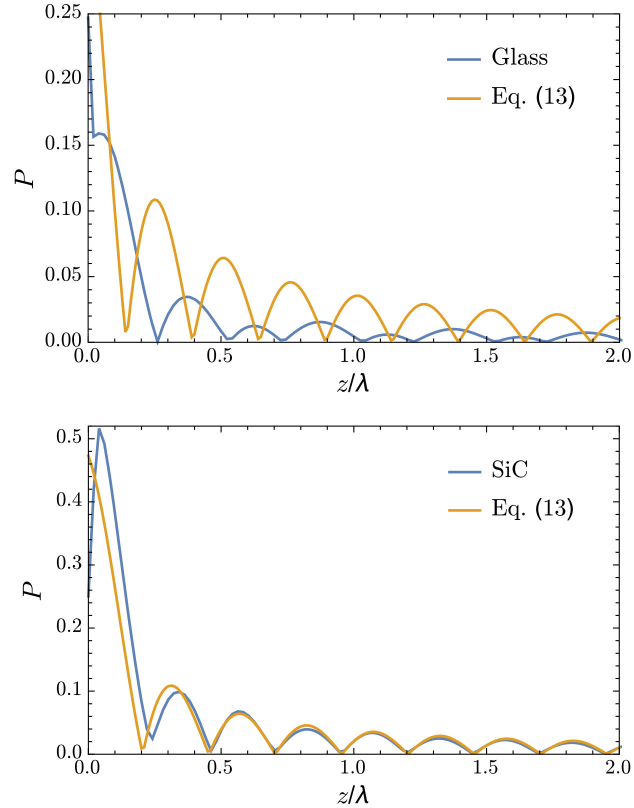

Fig. 3. Comparison of the asymptotic formula [Eq. (13)] with the exact result [Eq. (3)] for the degree of polarization.

4. DISCUSSION

We have investigated the polarization of near-field thermal emission. We find that, at thermal equilibrium, the degree of polarization exhibits spatial oscillations with a period of approximately $\lambda/2$, independent of material composition. The amplitude of the oscillations is greatest in systems with resonant surface waves such as surface-plasmons or surface-phonon polaritons.

In future work, we intend to explore the analogy between spatial correlations of near-field thermal radiation and near-field speckle patterns produced by volume scattering. It can be expected that thermal emission would correspond to a speckle pattern produced in transmission through a scattering medium, while thermal equilibrium would correspond to a speckle pattern produced by balanced illumination in reflection and transmission. Thus, the results presented herein could be relevant to the study of polarization in speckle patterns.

APPENDIX A

Here, we derive the asymptotic form of the degree of polarization given in Eq. (13). We assume that the real part of ϵ_1 is negative. We also assume that ϵ_1 has a small imaginary part; its real part is a large negative value, and the height z above the interface is much larger than the wavelength λ. That is, we suppose the following relations hold:

$$|\mathrm{Im}(\epsilon_1)| \ll |\mathrm{Re}(\epsilon_1)|, \quad \textbf{(A1)}$$

$$1 \ll \frac{4\pi z}{\lambda} \ll \sqrt{|\mathrm{Re}(\epsilon_1)|}. \tag{A2}$$

The assumption Eq. (A2) implies that

$$\zeta \ll \sqrt{|\mathrm{Re}(\epsilon_1)|}, \tag{A3}$$

where

$$\zeta = 4\pi\left(\frac{z}{\lambda} + \frac{1}{2\pi\sqrt{|\mathrm{Re}(\epsilon_1)|}}\right). \tag{A4}$$

Using Eq. (A1) we have

$$k_{1z} \sim \sqrt{k_0^2|\mathrm{Re}(\epsilon_1)| + q^2}. \tag{A5}$$

Let us introduce

$$t = \begin{cases} \sqrt{1 - \left(\frac{q}{k_0}\right)^2}, & 0 < q < k_0, \\ \sqrt{\left(\frac{q}{k_0}\right)^2 - 1}, & q > k_0. \end{cases} \tag{A6}$$

Noting that $|\mathrm{Re}(\epsilon_1)| \gg 1$, we can express k_{1z} as

$$k_{1z} \sim ik_0\sqrt{|\mathrm{Re}(\epsilon_1)|}\begin{cases} \left[1 + \frac{1-t^2}{2|\mathrm{Re}(\epsilon_1)|}\right], & 0 < q < k_0, \\ \left[1 + \frac{1+t^2}{2|\mathrm{Re}(\epsilon_1)|}\right], & q > k_0. \end{cases} \tag{A7}$$

Therefore, we obtain for $0 < q < k_0$

$$r_s \sim -1 - \frac{2it}{\sqrt{|\mathrm{Re}(\epsilon_1)|}}, \tag{A8}$$

$$r_p \sim \frac{|\mathrm{Re}(\epsilon_1)|}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\left[t^2 + \frac{2it}{\sqrt{|\mathrm{Re}(\epsilon_1)|}} - \frac{1}{|\mathrm{Re}(\epsilon_1)|}\right], \tag{A9}$$

and for $q > k_0$

$$\mathrm{Im}(r_s) \sim 0, \qquad \mathrm{Im}(r_p) \sim 0. \tag{A10}$$

Thus, we obtain

$$W_{11} = W_{22} \tag{A11}$$

$$\sim 2\pi k_0^2\left(\frac{k_0\hbar}{e^{\hbar\omega/k_BT} - 1} + \frac{k_0\hbar}{2}\right) \times \int_0^1 \left[1 + t^2 + \left(-2 + \frac{1}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\right)\cos(\zeta t) + t^2\left(-1 + \frac{2 + |\mathrm{Re}(\epsilon_1)|}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\right)\cos\left(\frac{4\pi z}{\lambda}t\right)\right]\mathrm{d}t, \tag{A12}$$

$$W_{33} \sim 4\pi k_0^2\left(\frac{k_0\hbar}{e^{\hbar\omega/k_BT} - 1} + \frac{k_0\hbar}{2}\right) \times \int_0^1 (1 - t^2)\left[1 + \frac{|\mathrm{Re}(\epsilon_1)|}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\cos(\zeta t) + \left(1 - \frac{2 + |\mathrm{Re}(\epsilon_1)|}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\right)\cos\left(\frac{4\pi z}{\lambda}t\right)\right]\mathrm{d}t. \tag{A13}$$

Taking Eq. (A3) into account, we can evaluate the integrals for W_{11} and W_{22} as follows:

$$\int_0^1 \left(-2 + \frac{1}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\right)\cos(\zeta t)\mathrm{d}t \sim -\frac{2}{\zeta}\sin\zeta + \frac{\pi}{2\sqrt{|\mathrm{Re}(\epsilon_1)|}}, \tag{A14}$$

$$\int_0^1 t^2\left(-1 + \frac{2 + a}{1 + at^2}\right)\cos(\zeta_0 t)\mathrm{d}t \sim -2\left(\frac{4\pi z}{\lambda}\right)^{-2}\cos\left(\frac{4\pi z}{\lambda}\right) + 2\left(\frac{4\pi z}{\lambda}\right)^{-3}\sin\left(\frac{4\pi z}{\lambda}\right) - \frac{\pi}{2\sqrt{|\mathrm{Re}(\epsilon_1)|}}. \tag{A15}$$

Similarly, for W_{33}, we have

$$\int_0^1 \frac{|\mathrm{Re}(\epsilon_1)|(1 - t^2)}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\cos(\zeta t)\mathrm{d}t \sim -\frac{1}{\zeta}\sin\zeta + \frac{\pi\sqrt{|\mathrm{Re}(\epsilon_1)|}}{2}, \tag{A16}$$

and

$$\int_0^1 (1 - t^2)\left(1 - \frac{2 + |\mathrm{Re}(\epsilon_1)|}{1 + |\mathrm{Re}(\epsilon_1)|t^2}\right)\cos\left(\frac{4\pi z}{\lambda}t\right)\mathrm{d}t \sim -2\left(\frac{4\pi z}{\lambda}\right)^{-2}\cos\left(\frac{4\pi z}{\lambda}\right) + \left[2 + \left(\frac{4\pi z}{\lambda}\right)^2\right]\left(\frac{4\pi z}{\lambda}\right)^{-3}\sin\left(\frac{4\pi z}{\lambda}\right) - \frac{\pi\sqrt{|\mathrm{Re}(\epsilon_1)|}}{2}. \tag{A17}$$

Using Eqs. (A2) and (A3), we arrive at

$$W_{11} = W_{22} \tag{A18}$$

$$\sim 2\pi k_0^2\left(\frac{k_0\hbar}{e^{\hbar\omega/k_BT} - 1} + \frac{k_0\hbar}{2}\right) \times \left[\frac{4}{3} - \frac{2}{\zeta}\sin\zeta - 2\left(\frac{4\pi z}{\lambda}\right)^{-2}\cos\left(\frac{4\pi z}{\lambda}\right)\right], \tag{A19}$$

$$W_{33} \sim 4\pi k_0^2\left(\frac{k_0\hbar}{e^{\hbar\omega/k_BT} - 1} + \frac{k_0\hbar}{2}\right) \times \left[\frac{2}{3} - 2\left(\frac{4\pi z}{\lambda}\right)^{-2}\cos\left(\frac{4\pi z}{\lambda}\right)\right]. \tag{A20}$$

Finally, we obtain

$$P \sim \frac{1}{2}\left|\frac{\sin\zeta}{\zeta}\right|, \tag{A21}$$

which is equivalent to Eq. (13). Evidently, this formula implies that the origin of polarization oscillations is due to the interplay of the numerator and denominator of 3.

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