Let us solve the following 2D boundary value problem with the Jacobi method.

$$\begin{aligned}
&\phi_{xx} + \phi_{yy} = 0, \qquad (x,y) \in (0,1) \times (0,1), \\
&\phi(x,1) = 1, \\
&\phi(x,0) = \phi(0,y) = \phi(1,y) = 0.
\end{aligned}$$

In the calculation the zero vector was chosen for the initial guess. The main part of the code is written as follows. As the stopping criterion, `tol=10e-4` was used.

```
while ratio>tol
  k=k+1;
  for i=2:n+1
  for j=2:n+1
    res(i,j)=(4*w(i,j)-w(i+1,j)-w(i-1,j)-w(i,j+1)-w(i,j-1))/(h^2);
  end
  end
  rn(k)=norm(res);
  ratio=rn(k)/rn(1);
  for i=2:n+1
  for j=2:n+1
    w_old(i,j)=(w(i+1,j)+w(i-1,j)+w(i,j+1)+w(i,j-1))/4;
  end
  end
  w=w_old;
end
```

The number of iterations k required for different methods is summarized as follows.

Jacobi

h	k	$\rho(B_J)$
1/4	26	0.7071
1/8	96	0.9239
1/16	334	0.9808

Gauss-Seidel

h	k	$\rho(B_{GS})$
1/4	15	0.5000
1/8	51	0.8536
1/16	172	0.9619

optimal SOR

h	k	$\rho(B_{\omega_*})$
1/4	9	0.1716
1/8	18	0.4465
1/16	34	0.6735

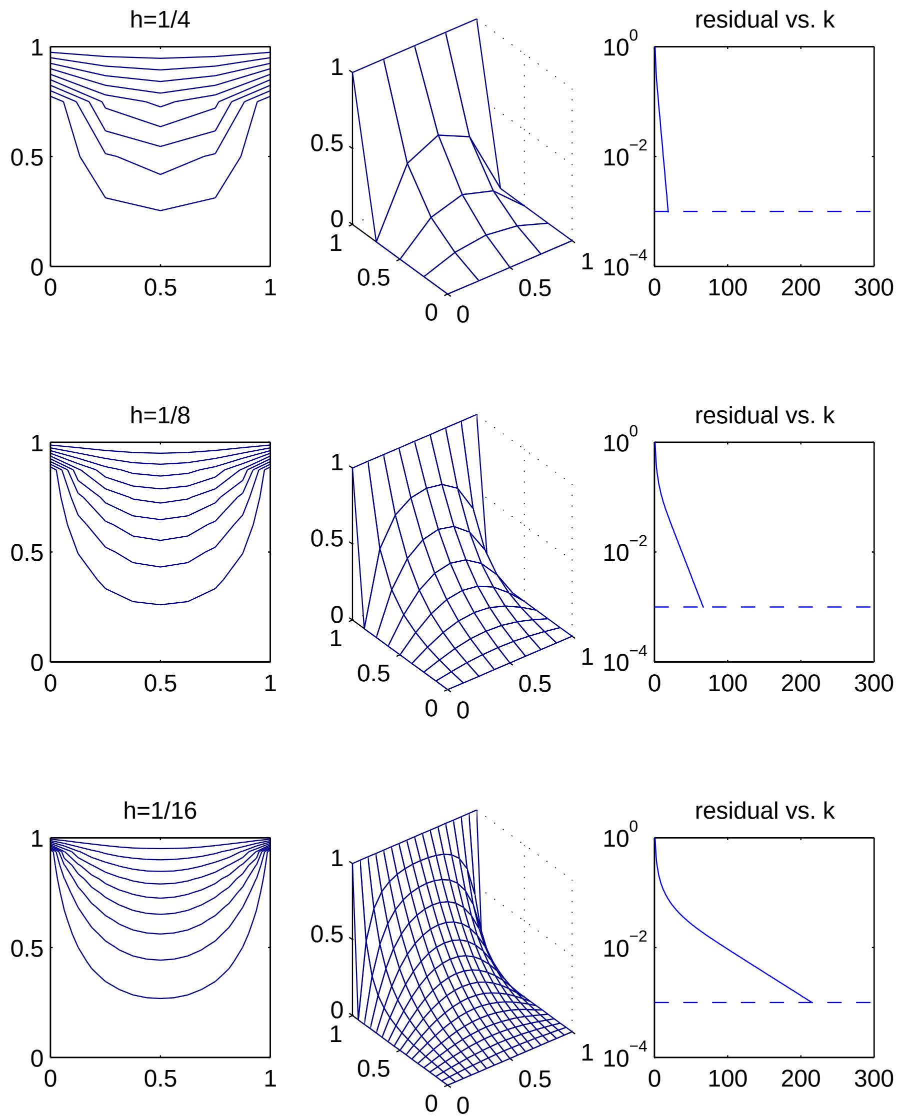

Fig. 2.1 Numerical solutions to $\phi_{xx} + \phi_{yy} = 0$, $\phi(x,1) = 1$, $\phi(x,0) = \phi(0,y) = \phi(1,y) = 0$. The Jacobi method is used.