The bisection method

Let us find a root of $f(x) = x^2 - 3$. We note that $f(1) = -2$ and $f(2) = 1$. Indeed, there is a root $r = \sqrt{3} = 1.73205\ldots$ on the interval $[1,2]$.

n	a_n	b_n	x_n	$f(x_n)$	$\lvert r - x_n \rvert$
0	1	2	1.5	−0.75	0.2321
1	1.5	2	1.75	0.0625	0.0179
2	1.5	1.75	1.625	−0.3594	0.1071
3	1.625	1.75	1.6875	−0.1523	0.0446
4	1.6875	1.75	1.71875	−0.0459	0.0133

Fixed-point iteration

To obtain the positive root of $f(x) = x^2 - 3 = 0$, we can rewrite the equation as

$$x = g_1(x) = \frac{3}{x}, \qquad x = g_2(x) = x - (x^2 - 3), \qquad x = g_3(x) = x - \frac{1}{2}(x^2 - 3).$$

Recall $r = \sqrt{3} = 1.73205\ldots$. Let us start the fixed-point iteration with $x_0 = 1.5$.

	Case 1	Case 2	Case 3
n	x_n	x_n	x_n
0	1.5	1.5	1.5
1	2	2.25	1.875
2	1.5	0.1875	1.6172
3	2	3.1523	1.8095
4	1.5	−3.7849	1.6723
5	2	−15.1106	1.7740

We see that Case 3 converges whereas Case 1 and Case 2 diverge.