# MATH 454 SECTION 002

## Quiz 11

April 18, 2014,  Instructor: Manabu Machida

Name: ______________________________________________

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- To receive full credit you must show all your work.
- You can use the back side of a paper if you need. Indicate where your calculation jumps.
- **NO CALCULATOR, SMARTPHONE, BOOKS, or OTHER NOTES.**

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Solve $tu_t + xu_x + 2u = 0$, $-\infty < x < \infty$, $1 < t$ with the initial condition $u(x,1) = \sin x$, $-\infty < x < \infty$. (*Hint:* $s = 0$ corresponds to the initial condition.)

**Solution [10]** Let us introduce $s$ and $\tau$ [2]. We have [1]

$$\begin{cases} \dfrac{dt}{ds} = t, & s > 0, \\ t = 1, & s = 0, \end{cases}$$

and [1]

$$\begin{cases} \dfrac{dx}{ds} = x, & s > 0, \\ x = \tau, & s = 0, \end{cases}$$

By solving the equations we obtain [2]

$$t = e^s, \qquad x = \tau e^s.$$

We note that

$$\frac{du}{ds} = \frac{\partial u}{\partial t}\frac{dt}{ds} + \frac{\partial u}{\partial x}\frac{dx}{ds} = tu_t + xu_x = -2u.$$

Therefore we have

$$\begin{cases} \dfrac{du}{ds} + 2u = 0, & s > 0, \\ u = \sin\tau, & s = 0. \end{cases}$$

We obtain [2]

$$u = e^{-2s}\sin\tau.$$

Since

$$s = \ln t, \qquad \tau = \frac{x}{t},$$

finally we obtain [2]

$$u(x,t) = \frac{1}{t^2}\sin\left(\frac{x}{t}\right).$$