MATH 454 SECTION 002

Quiz 10

April 11, 2014, Instructor: Manabu Machida

Name: ____

- To receive full credit you must show all your work.
- You can use the back side of a paper if you need. Indicate where your calculation jumps.
- **NO CALCULATOR, SMARTPHONE, BOOKS, or OTHER NOTES.**

Find the Fourier transform of $f(x)$, where $f(x) = e^{-(x-2)^2/2}$. [*Hint:* $f(x) = \int_{-\infty}^{\infty} \tilde{f}(\mu) e^{i\mu x} d\mu$ and $\tilde{f}(\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-i\mu x} dx$. Also $\int_{-\infty}^{\infty} e^{-a(x-b)^2} dx = \sqrt{\frac{\pi}{a}}$, $a > 0$.]

Solution [6] We will calculate [2]

$$\tilde{f}(\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-(x-2)^2/2} e^{-i\mu x} dx.$$

We have

$$\begin{aligned}\tilde{f}(\mu) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}x^2 + (2 - i\mu)x - 2\right] dx \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\Big(x - (2 - i\mu)\Big)^2 + \frac{(2 - i\mu)^2}{2} - 2\right] dx \\ &= \frac{1}{2\pi} \exp\left[\frac{-4i\mu - \mu^2}{2}\right] \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\Big(x - (2 - i\mu)\Big)^2\right] dx.\end{aligned}$$

Using the Gaussian integral we obtain [2]

$$\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\Big(x - (2 - i\mu)\Big)^2\right] dx = \sqrt{2\pi}.$$

Therefore the Fourier transform is obtained as [2]

$$\tilde{f}(\mu) = \frac{1}{2\pi} \exp\left[\frac{-4i\mu - \mu^2}{2}\right] \sqrt{2\pi} = \frac{1}{\sqrt{2\pi}} e^{-2i\mu} e^{-\mu^2/2}.$$