MATH 454 SECTION 002

Quiz 2

January 24, 2014, Instructor: Manabu Machida

Name: ____

- To receive full credit you must show all your work.
- You can use the back side of a paper if you need. Indicate where your calculation jumps.
- **NO CALCULATOR, SMARTPHONE, BOOKS, or OTHER NOTES.**

Find the Fourier sine series for $f(x) = e^x$, $0 < x < L$. [*Hint:* If necessary, you can use the formula $\int_0^L e^x \sin(cx)dx = \frac{1}{1+c^2}\left(c - ce^L\cos(cL) + e^L\sin(cL)\right)$.]

Solution [8] We extend $f(x)$ to the interval $-L < x < L$ by defining [2]

$$f_O(x) = \begin{cases} e^x & \text{for } 0 < x < L, \\ 0 & \text{for } x = 0, \\ -e^{-x} & \text{for } -L < x < 0. \end{cases}$$

We consider the Fourier series for this odd function f_O. Note that $A_0 = A_n = 0$ $(n = 1, 2, \ldots)$ [2]. Hence the Fourier series is written as

$$f_O(x) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}, \quad -L < x < L.$$

The coefficients B_n are given by [2]

$$B_n = \frac{1}{L}\int_{-L}^{L} f_O(x)\sin\frac{n\pi x}{L}dx = \frac{2}{L}\int_0^L e^x \sin\frac{n\pi x}{L}dx.$$

By doing the integral, we have

$$B_n = \frac{2}{L}\frac{1}{1+\left(\frac{n\pi}{L}\right)^2}\left[\frac{n\pi}{L} - \frac{n\pi}{L}e^L(-1)^n\right] = \frac{2\pi}{L^2}n\frac{1-e^L(-1)^n}{1+(n\pi/L)^2}.$$

Therefore the Fourier sine series is obtained as [2]

$$e^x = \frac{2\pi}{L^2}\sum_{n=1}^{\infty} n\frac{1-e^L(-1)^n}{1+(n\pi/L)^2}\sin\frac{n\pi x}{L}, \quad 0 < x < L.$$