Problem Set 5
Due on Fri, Aug 8

1) Find the determinants of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix}$.

2) Find the area of the triangle defined by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$.

3) Find the 3-volume of the 3-parallelepiped by $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

4) Let $\vec{v}$ be an eigenvector of A with associated eigenvalue λ. Is $\vec{v}$ an eigenvector of A^{-1}? If so, what is the eigenvalues?

5) Find a basis of the vector space V of all 2×2 matrices A for which $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ is an eigenvector, and thus determine the dimension of V.

6) Find all real eigenvalues, with their algebraic multiplicities.

   (a) $\begin{bmatrix} 0 & 4 \\ -1 & 4 \end{bmatrix}$, (b) $\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$, (c) $\begin{bmatrix} 2 & -2 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 2 & -3 \end{bmatrix}$.

7) Find all real eigenvalues. Then find a basis of each eigenspace, and find an eigenbasis, if it is possible.

   (a) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 2 & 2 & 0 \end{bmatrix}$.