Problem Set 4
Due on Fri, Aug 1

1) Consider the following vectors in $\mathbb{R}^4$.

$$\vec{u}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}.$$

Can you find a vector $\vec{u}_4$ in $\mathbb{R}^4$ such that the vectors $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$, $\vec{u}_4$ are orthonormal? If so, how many such vectors are there?

2) Find a basis for $W^{\perp}$, where

$$W = \text{span}(\begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}).$$

3) Two subspaces V and W of the same vector space are said to be orthogonal if $\vec{v} \cdot \vec{w} = 0$ for all $\vec{v} \in V$ and $\vec{w} \in W$. Answer True or False. Give a counter example if False.

   (a) $\ker(A)$ and $\text{im}(A)$ are orthogonal.

   (b) $\ker(A)$ and the row space (the span of rows of A) are orthogonal.

4) Perform the Gram-Schmidt process on the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 6 \\ 4 \end{bmatrix}$.

5) Find the least-squares solutions $\vec{x}^*$ of the system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}, \quad \text{and} \quad \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$