Problem Set 1
Due on Wed, Jul 2

1) Solve the following systems using Gauss-Jordan elimination.

(a) $\left| \begin{array}{r} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{array} \right|$, (b) $\left| \begin{array}{rrrrl} x_1 - 7x_2 & & & + \;\; x_5 & = 3 \\ & x_3 & & - 2x_5 & = 2 \\ & & x_4 + & x_5 & = 1 \end{array} \right|$,

(c) $\left| \begin{array}{r} x_1 + 2x_2 - 2x_3 + x_4 + 3x_5 = 2 \\ 2x_1 + \;\; x_2 + 2x_3 \qquad + \;\; x_5 = 3 \\ -2x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 = 1 \end{array} \right|$.

2) Determine which of the matrices below are in reduced row-echelon form.

(a) $\begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, (b) $\begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$,

(d) $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$.

3) Find the rank of the matrices. (a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$.

4) Find a matrix A of rank 1 such that $A \begin{bmatrix} 5 \\ 3 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

5) Let $\vec{x}_1$ be a solution of $A\vec{x} = \vec{b}$. Justify (a) and (b):

   (a) If $\vec{x}_h$ is a solution of $A\vec{x} = \vec{0}$, then $\vec{x}_1 + \vec{x}_h$ is a solution of $A\vec{x} = \vec{b}$.

   (b) If $\vec{x}_2$ is another solution of $A\vec{x} = \vec{b}$, then $\vec{x}_2 - \vec{x}_1$ is a solution of $A\vec{x} = \vec{0}$.

6) Find all lower triangular 3×3 matrices X such that X^3 is the zero matrix.